which is comparable to the operating RF period. As Bady admits in his rebuttal, "T is frequency and structure sensitive and must be determined experimentally." Therefore, M is frequency dependent at millimeter-wavelengths. Then the anistrophy field[8]

$$H_a = \frac{-2(K_1 + 2K_2)}{M} \text{ or } \frac{2K_1}{M} \qquad (2)$$

(depending on the direction of the anisotrophy field) is also frequency dependent because M appears in (2). Using Kittel's relation,

$$\omega_r^2 = [\omega_0+(N_x-N_z)\omega_M][\omega_0+(N_y-N_z)\omega_M]$$

$$= \gamma^2[H_0+H_a]^2 \qquad (3)$$

$$H_a = \frac{\sqrt{[\omega_0+(N_x-N_z)\omega_M][\omega_0+(N_y-N_z)\omega_M]}}{\gamma} - H_0 \qquad (4)$$

the frequency dependence of H_a is also obvious because (4) contains N_x, N_y, N_z and $\omega_M = 4\pi\gamma M$. For these reasons, the authors stated in the previous letter that M and H_a are not *accurately* known at millimeter-wavelengths.

8) In microwaves, it is possible to consider that, at a frequency which is low enough so that the phenomenalogical relaxation time T is small compared with the period of the operating microwave frequency, the quantity found by the magnetostatic method may be usable. In fact, Kittel,[12] and J. Smit and H. G. Beljers,[13] showed experimentally that the quantity found in the static method is applicable in the range of microwave frequencies.

9) The values of M, 380 gauss and H_a, 17000 oersted given by Smit and Wijn[8] were obtained by the static method in a specified direction of magnetization and not at millimeter wavelengths. Validity of these values at millimeter-wavelengths in the range of 3 mm is questionable for the reasons mentioned above. The application of these quantities to the authors' complicated isolators is even more questionable. The value of H_a at the 5 mm wave-length range[14] was estimated to be 18,400 oersted for a single crystal of $BaFe_{12}O_{19}$ of density 5.13 g/cm^3 which is 97 per cent of the true X-ray density.

The authors sincerely appreciate Bady's interesting questions.

I. Bady[15]

This writer continues to disagree with many statements made by Vilmur and Ishii, but for the sake of brevity will make only one short comment. Vilmur and Ishii make several remarks such as, "Bady does not give any theoretical reason why the large thickness made R low," " . . . it is impossible to explain experimental results using Bady's simplified relations." I have clearly stated in my rebuttal that the large thickness of the sample greatly distorts the RF fields in the sample (as compared to what the fields would be in an empty waveguide) and this makes perturbation theory inapplicable. Hence R cannot be calculated by Lax's formula, as done by Vilmur and Ishii, since the formula is based on perturbation theory. Also I have clearly stated in my original comments that I am not surprised that the simple formula for linewidth does not fit experimental data, since the large sample thickness makes perturbation theory inapplicable and anomolous results may occur.

P. Vilmur and K. Ishii[16]

1) Bady's explanations are qualitative in nature. What the authors want is an exact quantitative theoretical proof to support Bady's conclusion.

2) Inapplicability of the perturbation theory may not guarantee low value of R.

3) Bady states that the large sample makes his formulas inapplicable. This implies that if the sample is made smaller, the sample will follow Bady's simplified formula. Here is a problem to be cleared in Bady's approach. If the sample is made smaller, the sample will follow more exactly Lax's formula instead of Bady's simplified formula, because, as Bady has been asserting, the perturbation theory is applicable with less errors for smaller samples.

[12] C. Kittel, "Interpretation of anomalous Larmor frequencies in ferromagnetic resonance experiment," *Phys. Rev.*, vol. 71, pp. 270–271; February, 1947.

[13] J. Smit and H. G. Beljers, "Ferromagnetic resonance absorption in $BaFe_{12}O_{19}$, a highly anisotropic crystal," *Philips Res. Rept.*, vol. 10, pp. 113–130; 1955.

[14] F. F. Y. Wang, K. Ishii, J. B. Y. Tsui "Ferrimagnetic resonance of single-crystal barium ferrite in the millimeter-wave region," *J. Appl. Phys.*, vof. 32, pp. 1621–1622; August, 1961.

[15] Received May 21, 1963.

[16] Received June 13, 1963.

Ferromagnetic Line Width of Nonoriented Polycrystalline Hexagonal Ferrites with Large Magnetic Anisotropy Fields*

Introduction

Data on the line widths of oriented polycrystalline, hexagonal ferrites with large magnetic anisotropy fields have shown that the uniaxial ferrites (easy direction of magnetization along the C axis) have a considerably larger line width than that of planar ferrites (easy plane of magnetization perpendicular to the C axis). For example, in work performed at Philips[1] on the uniaxial barium and strontium ferrites of the magnetoplumbite structure, with aluminum or titanium-cobalt substitutions, the line width varied over a range of 1600 to 3300 oersteds for materials with anisotropies ranging from 7000 to 52,000 oersteds. There was no strong correlation between line width and anisotropy field. In work done at Sperry[2] on uniaxial nickel-W compounds with cobalt substitutions, the line width ranged from 2200 to 3000 oersteds for materials with anisotropies ranging from 7000 to 12,800 oersteds. On the other hand, in work performed by RCA on planar ferrites, a line width as low as 110 oersteds was obtained,[3] and a large number of compounds had a line width less than 500 oersteds.[4]

It is very unlikely that the large line width of polycrystalline uniaxial ferrites is due to the crystallite's line width Though relatively little work has been done on single crystals of hexagonal ferrites, a line width of 50 oersteds has been obtained on a single crystal of barium ferrite[5] and on a single crystal of aluminum substituted strontium ferrite.[6] A line width of 18 oersteds was obtained on a single crystal of the planar ferrite Zn_2Y.[7] However, there has been considerably more research done on single crystals of Zn_2Y ferrite than on those of uniaxial ferrites, to reduce line width.

A major contribution to the line width of oriented hexagonal ferrites, both of uniaxial and planar types, was considered to be imperfect orientation. It was therefore desirable to study the extreme case of imperfect orientation, *i.e.*, completely nonoriented materials, and compare the theoretically calculated line widths of the uniaxial and planar ferrites for this case.

Method of Calculation

Only a brief outline of the method used to calculate the line widths of the nonoriented uniaxial and planar ferrites will be given in this communication. More details are contained in a Technical Report[8] with the same title published by the United States Army Electronic Research and Development Laboratory.

The nonoriented ferrite was assumed to be composed of small, single domain crystallites whose C axes were randomly oriented over all possible solid angles. It was further assumed that the crystallites

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[1] D. J. DeBitetto, F. K. duPré, and F. G. Brockman, "Hexagonal Magnetic Materials for Microwave Applications," Philips Laboratories, Irvington-on-Hudson, N. Y., Final Rept., Contract DA36-039 SC-85279; July, 1961.

[2] G. Rodrique and J. Pippin, "Theoretical and Experimental Investigation to Determine the Microwave Characteristics and Applications of Hexagonal Magnetic Oxides to Microwave Circuitry," Sperry Microwave Electronics Co., Clearwater, Fla., Tech. Rept. Contract AF30 (602) 2330; December, 1961.

[3] R. Harvey, I. Gordon, and R. Braden, "Hexagonal Magnetic Compounds," RCA Laboratories, Princeton, N. J. Quarterly Rept. No. 6, Contract DA36-039 SC-87433; December, 1962.

[4] R. Harvey, I. Gordon, and R. Braden, "Hexagonal Magnetic Compounds," RCA Laboratories, Princeton, N. J , Final Rept., Contract DA36-039 SC-78288; June, 1961.

[5] I. Bady, T. Collins, D. J. DeBitetto, and F. K. duPré, "Ferromagnetic linewidth of single crystals of barium ferrite ($BaFe_{12}O_{19}$)," Proc. IRE (*Correspondence*), vol. 58, p. 2033; December, 1960.

[6] D. J. DeBitello, F. K. duPré, and F. G. Brockman, "Hexagonal Magnetic Materials for Microwave Applications," Philips Laboratories, Irvington-on-Hudson, N. Y., Final Rept., Contract DA36-039 SC-78071; July, 1961.

[7] A. Tauber, R. Savage, R. Gambino, and C. Whinfrey, "Growth of single crystal hexagonal ferrites containing Zn," *J. Appl. Phys.*, Suppl. to vol. 33, pp. 1381–1382, March, 1962.

[8] I. Bady and G. McCall, "Linewidth of nonoriented polycrystalline hexagonal ferrites with large magnetic anisotropy fields," U. S. Army Electronic Research and Development Laboratory, Fort Monmouth, N. J., Technical Rept. 2350,; March, 1963.

did not interact with each other. Demagnetizing factors were disregarded for the sake of simplicity.

Let us consider a resonant cavity containing the nonoriented ferrite. A biasing field is applied in a direction perpendicular to the RF magnetic field in the cavity. The resonant frequency of each crystallite will be determined by its anisotropy field, the biasing field, and the angle ψ its C axis makes with the biasing field. At one particular angle ψ_r for a given biasing field, the resonant frequency of the crystallite will be exactly the same as the test frequency and have the maximum interaction with the cavity. As the angle of the C axis departs from ψ_r, the resonant frequency becomes increasingly different from the test frequency and the interaction with the cavity decreases. We calculate the angles ψ_1, and ψ_2 between which a crystallite must lie in order that its resonant frequency will differ from the test frequency by no more than a chosen amount. All crystallites within this angle are presumed to absorb energy equally; all other crystallites are presumed not to absorb any energy. Let Ω be the solid angle subtended between the cones defined by ψ_1 and ψ_2. The loss term of magnetic susceptibility is proportional to Ω and therefore a plot of Ω vs biasing field is a plot of the relative value of χ'', the loss term of the susceptibility, vs biasing field. The line width is readily determined from such a curve.

Discussion

A plot of χ'' (relative) for a nonoriented uniaxial ferrite is shown in Fig. 1, and plots for nonoriented planar ferrites are shown in Fig. 2. The abscissa in both figures is the shifted biasing field $H_0 - H_r$ where H_0 is the applied biasing field and H_r is the biasing field required for ferromagnetic resonance for a crystallite whose easy direction is parallel to the biasing field (for the uniaxial ferrite) or whose easy plane is parallel to the biasing field (for the planar ferrite). Ha is the magnetic anistropy field.

A comparison of Figs. 1 and 2 shows that the line width of the nonoriented uniaxial ferrite is indeed very much larger than that of the planar ferrite. The most suitable comparison is between curve I of Fig. 2 and the curve in Fig. 1, since both have approximately the same value of anisotropy field and the same value of H_r. We note that the line width of the uniaxial ferrite is almost five times that of the planar ferrite.

The relatively narrow line width of nonoriented planar ferrites has been confirmed experimentally. Schlömann[9] has reported a line width of 500 oersteds for a nonoriented zinc Y. Of six nonoriented planar ferrites measured here, three had line widths of 1500 oersteds or less. It is interesting to find that completely nonoriented planar ferrites can have line widths narrower than the narrowest line width that has up to now been obtained with oriented polycrystalline uniaxial ferrites.

[9] E. Schlömann and R. Jones, "Ferromagnetic resonance in polycrystalline ferrites with hexagonal crystal structure," *J. Appl. Phys.*, Suppl. to vol. 30, pp. 177–178; April, 1959.

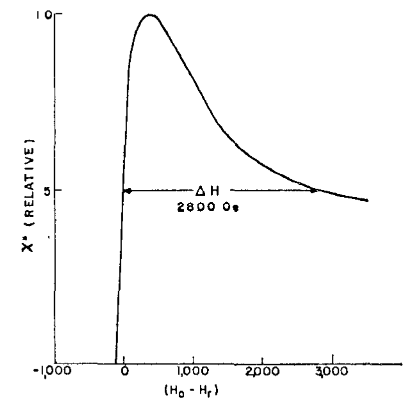

Fig. 1—Plot of x'' (relative) vs shifted biasing field for a uniaxial ferrite with $H_a = 8500$ oersteds, $\omega/\gamma = 10{,}500$ oersteds, $H_r = 2000$ oersteds.

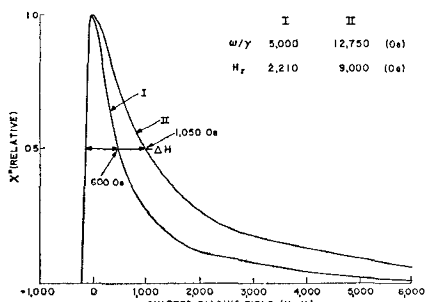

Fig. 2—Plot of χ'' (relative) vs shifted biasing field for planar ferrite with $H_a = 9000$ oersteds, $\omega/\gamma = 5000$ and 12,750 oersteds.

An understanding as to why the line width of nonoriented uniaxial ferrites is so much greater than that of nonoriented planar ferrites can be obtained from the following reasoning. The magnitude of Ω, and hence the magnitude of the loss term of the susceptibility, is proportional to two factors. Factor 1 is the magnitude of $|\psi_1 - \psi_2|$ and factor 2 is the solid angle, Ω_Δ subtended between the cones defined by ψ_r and $\psi_r + \Delta\psi_r$ where $\Delta\psi_r$ is a small increase in ψ_r. The terms Ω, ψ_r, ψ_1 and ψ_2, have been defined previously.

Let us consider the variation of the two factors as a function of biasing field. Factor 1 is maximum when the biasing field is such that crystallites that are at ferromagnetic resonance are those whose easy direction of magnetization, or easy plane of magnetization (as applicable) are parallel to the biasing field. This biasing field has previously been designated as H_r. Factor 1 decreases as the biasing field is increased beyond H_r. Thus factor 1 is relatively large when ψ is close to 0° for the uniaxial ferrites, and close to 90° for the planar ferrites.

The solid angle subtended between the cones defined by ψ_r and $\psi_r + \Delta\psi_r$ is proportional to sin ψ_r. Thus factor 2 is small for biasing fields close to H_r for uniaxial ferrites and increases as the biasing field is increased beyond H_r. In the case of planar ferrites, factor 2 is large for biasing fields close to H_r and decreases as the biasing field is increased beyond H_r.

Thus in the case of uniaxial ferrites, as the biasing field is increased beyond H_r, factor 1 decreases and factor 2 increases. This tends to reduce the dependence of Ω on H_r as the biasing field is increased beyond H_r and results in a relatively broad line width. In the case of the planar ferrites, however, both factors are large in the vicinity of H_r, and both decrease as the biasing field is increased beyond H_r. Thus, there is a relatively sharp peak of Ω in the vicinity of H_r, and this results in a relatively narrow line width.

Conclusions

Theoretical calculations show that nonoriented uniaxial ferrites have a much wider line width than that of nonoriented planar ferrites. Thus the imperfect orientation that will inevitably occur when processing oriented polycrystalline hexagonal ferrites, will have a greater effect on broadening the line width of uniaxial ferrites than that of planar ferrites. This explains at least part of the reason why oriented planar ferrites generally have a much narrower line width than that of oriented uniaxial ferrites. In fact, a number of completely nonoriented planar ferrites have been prepared which have a substantially narrower line width than the narrowest line width achieved so far with polycrystalline oriented uniaxial ferrites.

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E-Plane 3-Port X-Band Waveguide Circulators*

When a circulator is used with a parametric amplifier or maser, the noise contribution of the circulator may be reduced by cooling it in liquid nitrogen or liquid helium. Compact devices are required to put in the dewar and, depending on the microwave frequency, a compromise may be necessary in choosing between a compact stripline circulator and a comparatively bulky H-plane waveguide circulator, because waveguide feeds will have lower loss than coaxial line. This problem may be eased by using a very compact E-plane waveguide circulator, as shown in Fig. 1(a).

It can be shown that the circulation bandwidth and performance of a lossless, nonreciprocal, symmetrical 3-port waveguide junction are dependent only on the frequency characteristic of the reflection coefficient. In a practical device the circulation may occur in opposite senses at various frequencies. These modes of circulation can be defined in terms of the microwave frequency, the value of applied magnetic field, the direction of circulation and the dimensions of the ferrite. Therefore, in principle, there are two stages in the development of a

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